

Physics 4A

Chapters 12: Rotation of a Rigid Body

GENERAL PRINCIPLES

Solving Rotational Dynamics Problems

MODEL Model the object as a **rigid body**.

VISUALIZE Draw a pictorial representation.

SOLVE Use **Newton's second law** for rotational motion:

$$\alpha = \frac{\tau_{\text{net}}}{I}$$

Use rotational kinematics to find angles and angular velocities.

ASSESS Is the result reasonable?

Conservation Laws

Energy is conserved for an isolated system.

- Pure rotation $E = K_{\text{rot}} + U_G = \frac{1}{2}I\omega^2 + Mgy_{\text{cm}}$

- Rolling $E = K_{\text{rot}} + K_{\text{cm}} + U_G = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv_{\text{cm}}^2 + Mgy_{\text{cm}}$

Angular momentum is conserved if $\vec{\tau}_{\text{net}} = \vec{0}$.

- Particle $\vec{L} = \vec{r} \times \vec{p}$

- Rotation about a symmetry axis or fixed axle $\vec{L} = I\vec{\omega}$

© 2017 Pearson Education, Inc.

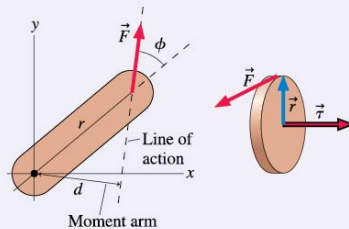
IMPORTANT CONCEPTS

Torque is the rotational equivalent of force:

$$\tau = rF \sin \phi = rF_t = dF$$

The vector description of torque is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

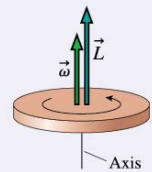


Vector description of rotation

Angular velocity $\vec{\omega}$ points along the rotation axis in the direction of the right-hand rule.

For a rigid body rotating about a fixed axle or an axis of symmetry, the angular momentum is $\vec{L} = I\vec{\omega}$.

Newton's second law is $\frac{d\vec{L}}{dt} = \vec{\tau}_{\text{net}}$.



A system of particles on which there is no net force undergoes unconstrained rotation about the **center of mass**:

$$x_{\text{cm}} = \frac{1}{M} \int x \, dm \quad y_{\text{cm}} = \frac{1}{M} \int y \, dm$$

The gravitational torque on a body can be found by treating the body as a particle with all the mass M concentrated at the center of mass.

The **moment of inertia**

$$I = \sum_i m_i r_i^2 = \int r^2 \, dm$$

is the rotational equivalent of mass. The moment of inertia depends on how the mass is distributed around the axis. If I_{cm} is known, I about a parallel axis distance d away is given by the **parallel-axis theorem**: $I = I_{\text{cm}} + Md^2$.

© 2017 Pearson Education, Inc.

APPLICATIONS

Rigid-body model

- Size and shape do not change as the object moves.
- The object is modeled as particle-like atoms connected by massless, rigid rods.



Rigid-body equilibrium

An object is in total equilibrium only if both $\vec{F}_{\text{net}} = \vec{0}$ and $\vec{\tau}_{\text{net}} = \vec{0}$.

No rotational or translational motion

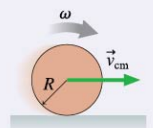


Rolling motion

For an object that rolls without slipping

$$v_{\text{cm}} = R\omega$$

$$K = K_{\text{rot}} + K_{\text{cm}}$$

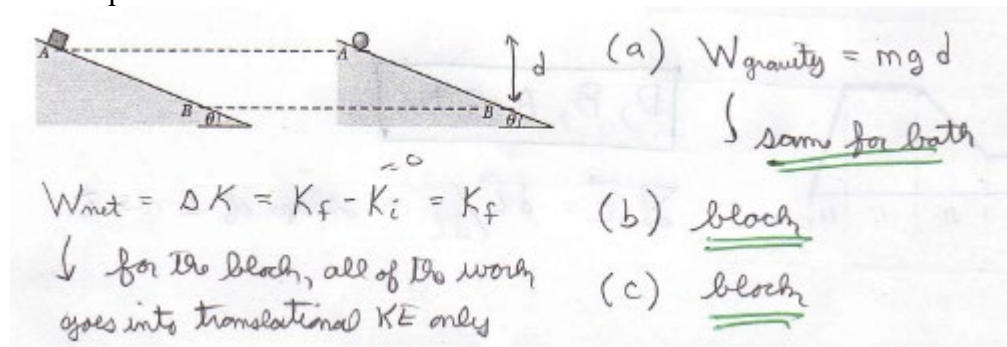


© 2017 Pearson Education, Inc.

Question and Example Problems from Chapter 12

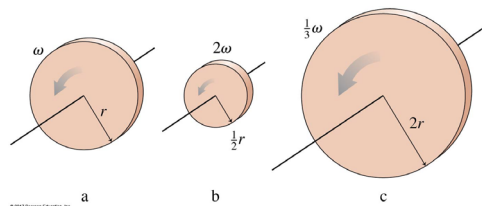
Conceptual Question 12-A

In the figure below, a block slides down a frictionless ramp and a sphere rolls without sliding down a ramp of the same angle θ . The block and sphere have the same mass, start from rest at point A, and descend through point B. (a) In that descent, is the work done by the gravitational force on the block greater than, less than, or the same as the work done by the gravitational force on the sphere? At B, which object has more (b) translational kinetic energy and (c) speed down the ramp?



Conceptual Question 12-3

The figure shows three rotating disks, all of equal mass. Rank, in order, from largest to smallest, their rotational kinetic energies K_a to K_c .



12.3. The rotational kinetic energy is $K_{\text{rot}} = \frac{1}{2}I\omega^2$. For a disk, $I = \frac{1}{2}MR^2$. Since the mass is the same for all three disks, the quantity $R^2\omega^2$ determines the ranking. Thus $K_a = K_b > K_c$.

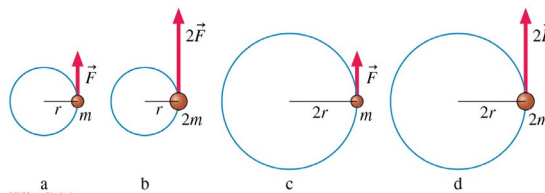
Conceptual Question 12-7

The professor hands you two spheres. They have the same mass, the same radius, and the same exterior surface. The professor claims one is a solid sphere and the other is hollow. Can you determine which is which without cutting them open? If so, how? If not, why not?

12.7. It will be easier to rotate the solid sphere because the hollow sphere's mass is generally distributed farther from its center. If you roll both simultaneously down an incline, the solid sphere will win.

Conceptual Question 12-10

Rank in order, from largest to smallest, the angular accelerations α_a to α_d in the figure. *Explain.*



12.10. Since $\tau = I\alpha$, $\alpha = \frac{\tau}{I}$. Also, $\tau = Fr$ and $I = mr^2 \Rightarrow \alpha = \frac{F}{mr}$. Calculate α for each case:

$$\alpha_a = \frac{F_0}{m_0 r_0} \qquad \alpha_c = \frac{F_0}{m_0 (2r_0)} = \frac{1}{2} \alpha_a$$

$$\alpha_b = \frac{2F_0}{2m_0r_0} = \alpha_a$$

$$\alpha_d = \frac{2F_0}{(2m_0)(2r_0)} = \frac{1}{2}\alpha_a$$

So $\alpha_a = \alpha_b > \alpha_c = \alpha_d$.

Problem 4-41

An electric fan goes from rest to 1800 rpm in 4.0 s. What is the angular acceleration?

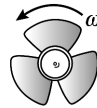
4.41. Model: The fan is in nonuniform circular motion.

Visualize:

Pictorial representation

Known
 $\omega_i = 0$
 $\omega_f = 1800 \text{ rpm}$
 $\Delta t = 4 \text{ s}$

Find
 α



Solve: Note $1800 \text{ rev/min} \left(\frac{\text{min}}{60 \text{ s}} \right) = 30 \text{ rev/s}$. Thus $\omega_f = \omega_i + \alpha\Delta t \Rightarrow 30 \text{ rev/s} = 0 \text{ rev/s} + \alpha(4.0 \text{ s}) \Rightarrow \alpha = 7.5 \text{ rev/s}^2$.

This can be expressed as $(7.5 \text{ rev/s}) \left(\frac{2\pi \text{ rad}}{\text{rev}} \right) = 47 \text{ rad/s}^2$.

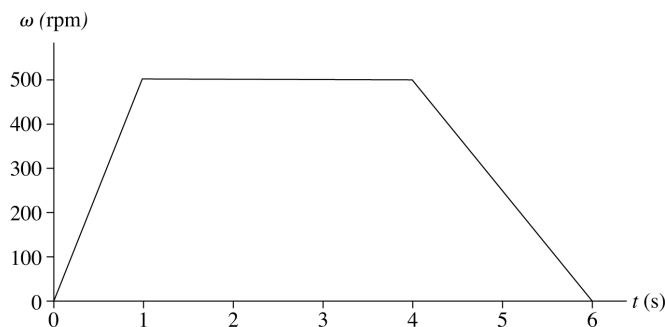
Assess: An increase in the angular velocity of a fan blade by 7.5 rev/s each second seems reasonable.

Problem 4-43

Starting from rest, a DVD steadily accelerates to 500 rpm in 1.0 s, rotates at this angular speed for 3.0 s, then steadily decelerates to a halt in 2.0 s. How many revolutions does it make?

4.43. Model: The DVD is a rotating rigid body.

Visualize: The angular displacement (angle turned) is the area under the angular velocity graph.



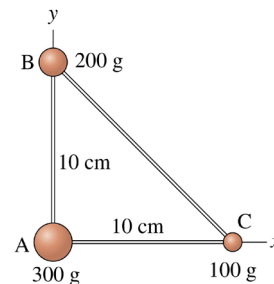
Solve:

$$\Delta\theta = \frac{1}{2}(1 \text{ s})(500 \text{ rpm}) + (3 \text{ s})(500 \text{ rpm}) + \frac{1}{2}(2.0 \text{ s})(500 \text{ rpm}) = (4.5 \text{ s}) \left(500 \frac{\text{rev}}{\text{min}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = 38 \text{ rev}$$

Assess: 38 revolutions seem like a reasonable amount in 6 seconds.

Problem 12-6

The three masses shown in the figure are connected by massless, rigid rods. What are the coordinates of the center of mass?



12.6. Visualize: The coordinates of the three masses m_A , m_B , and m_C are (0 cm, 0 cm), (0 cm, 10 cm), and (10 cm, 0 cm), respectively.

Solve: The coordinates of the center of mass are

$$x_{\text{cm}} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C} = \frac{(300 \text{ g})(0 \text{ cm}) + (200 \text{ g})(0 \text{ cm}) + (100 \text{ g})(10 \text{ cm})}{(300 \text{ g} + 200 \text{ g} + 100 \text{ g})} = 1.7 \text{ cm}$$

$$y_{\text{cm}} = \frac{m_A y_A + m_B y_B + m_C y_C}{m_A + m_B + m_C} = \frac{(300 \text{ g})(0 \text{ cm}) + (200 \text{ g})(10 \text{ cm}) + (100 \text{ g})(0 \text{ cm})}{(300 \text{ g} + 200 \text{ g} + 100 \text{ g})} = 3.3 \text{ cm}$$

Problem 12-10

What is the rotational kinetic energy of the earth? Assume the earth is a uniform sphere. Data for the earth can be found inside the back cover of the book.

12.10. Model: The earth is a rigid, spherical rotating body.

Solve: The rotational kinetic energy of the earth is $K_{\text{rot}} = \frac{1}{2} I \omega^2$. The moment of inertia of a sphere about its diameter (see Table 12.2) is $I = \frac{2}{5} M_{\text{earth}} R^2$ and the angular velocity of the earth is

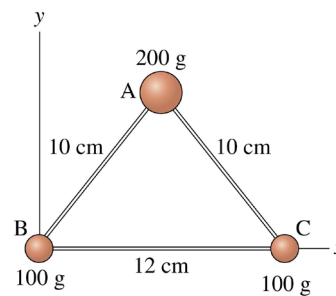
$$\omega = \frac{2\pi \text{ rad}}{24 \times 3600 \text{ s}} = 7.27 \times 10^{-5} \text{ rad/s}$$

Thus, the rotational kinetic energy is

$$K_{\text{rot}} = \frac{1}{2} \left(\frac{2}{5} M_{\text{earth}} R^2 \right) \omega^2 = \frac{1}{5} (5.98 \times 10^{24} \text{ kg})(6.37 \times 10^6 \text{ m})^2 (7.27 \times 10^{-5} \text{ rad/s})^2 = 2.57 \times 10^{29} \text{ J}$$

Problem 12-15

The three masses shown in the figure are connected by massless, rigid rods. **(a)** Find the coordinates of the center of mass. **(b)** Find the moment of inertia about an axis that passes through mass A and is perpendicular to the page. **(c)** Find the moment of inertia about an axis that passes through masses B and C.



12.15. Model: The three masses connected by massless rigid rods are a rigid body.

Solve: **(a)** $x_{\text{cm}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(0.100 \text{ kg})(0 \text{ m}) + (0.200 \text{ kg})(0.06 \text{ m}) + (0.100 \text{ kg})(0.12 \text{ m})}{0.100 \text{ kg} + 0.200 \text{ kg} + 0.100 \text{ kg}} = 0.060 \text{ m}$

$$y_{\text{cm}} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(0.100 \text{ kg})(0 \text{ m}) + (0.200 \text{ kg}) \left(\sqrt{(0.10 \text{ m})^2 - (0.06 \text{ m})^2} \right) + (0.100 \text{ kg})(0 \text{ m})}{0.100 \text{ kg} + 0.200 \text{ kg} + 0.100 \text{ kg}} = 0.040 \text{ m}$$

(b) The moment of inertia about an axis through A and perpendicular to the page is

$$I_A = \sum m_i r_i^2 = m_B (0.10 \text{ m})^2 + m_C (0.10 \text{ m})^2 = (0.100 \text{ kg})[(0.10 \text{ m})^2 + (0.10 \text{ m})^2] = 0.0020 \text{ kg m}^2$$

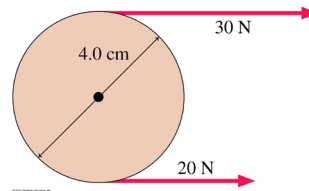
(c) The moment of inertia about an axis that passes through B and C is

$$I_{BC} = m_A \left(\sqrt{(0.10 \text{ m})^2 - (0.06 \text{ m})^2} \right)^2 = 0.00128 \text{ kg m}^2 \approx 0.0013 \text{ kg m}^2$$

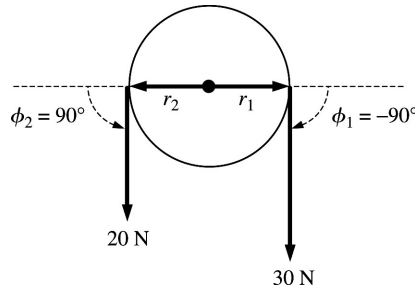
Assess: Note that mass m_A does not contribute to I_A , and the masses m_B and m_C do not contribute to I_{BC} .

Problem 12-18

In the figure, what is the net torque about the axle?



12.18. Visualize:



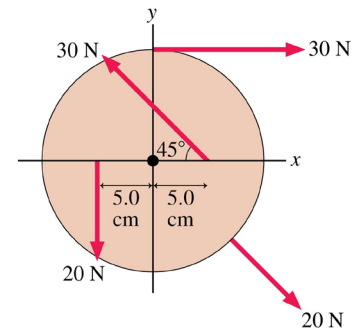
Solve: Torque by a force is defined as $\tau = Fr \sin \phi$ where ϕ is measured counterclockwise from the \vec{r} vector to the \vec{F} vector. The net torque on the pulley about the axle is the torque due to the 30 N force plus the torque due to the 20 N force:

$$\begin{aligned} (30 \text{ N})r_1 \sin \phi_1 + (20 \text{ N})r_2 \sin \phi_2 &= (30 \text{ N})(0.02 \text{ m}) \sin(-90^\circ) + (20 \text{ N})(0.02 \text{ m}) \sin(90^\circ) \\ &= (-0.60 \text{ N} \cdot \text{m}) + (0.40 \text{ N} \cdot \text{m}) = -0.20 \text{ N} \cdot \text{m} \end{aligned}$$

Assess: A negative torque causes a clockwise acceleration of the pulley.

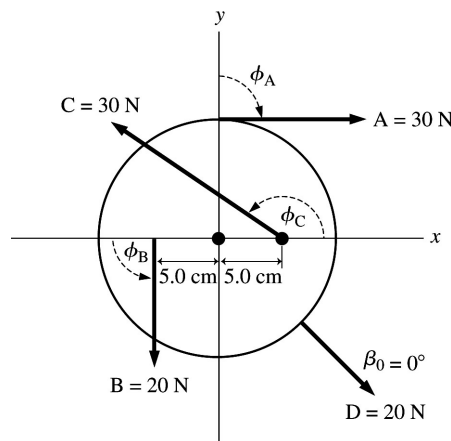
Problem 12-20

The 20-cm-diameter disk in the figure can rotate on an axle through its center. What is the net torque about the axle?



12.20. Model: The disk is a rotating rigid body.

Visualize:



The radius of the disk is 10 cm and the disk rotates on an axle through its center.

Solve: The net torque on the axle is

$$\begin{aligned} \tau &= F_A r_A \sin \phi_A + F_B r_B \sin \phi_B + F_C r_C \sin \phi_C + F_D r_D \sin \phi_D \\ &= (30 \text{ N})(0.10 \text{ m}) \sin(-90^\circ) + (20 \text{ N})(0.050 \text{ m}) \sin 90^\circ + (30 \text{ N})(0.050 \text{ m}) \sin 135^\circ + (20 \text{ N})(0.10 \text{ m}) \sin 0^\circ \\ &= -3 \text{ N m} + 1 \text{ N m} + 1.0607 \text{ N m} = -0.94 \text{ N m} \end{aligned}$$

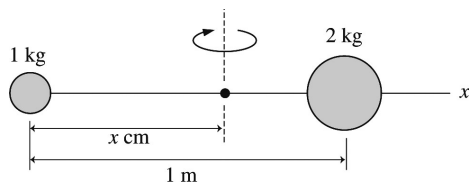
Assess: A negative torque means a clockwise rotation of the disk.

Problem 12-26

A 1.0 kg ball and a 2.0 kg ball are connected by a 1.0-m-long rigid, massless rod. The rod is rotating CW about its center of mass at 20 rpm. What net torque will bring the balls to a halt in 5.0 s?

12.26. Model: Two balls connected by a rigid, massless rod are a rigid body rotating about an axis through the center of mass. Assume that the size of the balls is small compared to 1 m.

Visualize:



We placed the origin of the coordinate system on the 1.0 kg ball.

Solve: The center of mass and the moment of inertia are

$$x_{\text{cm}} = \frac{(1.0 \text{ kg})(0 \text{ m}) + (2.0 \text{ kg})(1.0 \text{ m})}{(1.0 \text{ kg} + 2.0 \text{ kg})} = 0.667 \text{ m} \quad \text{and} \quad y_{\text{cm}} = 0 \text{ m}$$

$$I_{\text{about cm}} = \sum m_i r_i^2 = (1.0 \text{ kg})(0.667 \text{ m})^2 + (2.0 \text{ kg})(0.333 \text{ m})^2 = 0.667 \text{ kg m}^2$$

We have $\omega_f = 0 \text{ rad/s}$, $t_f - t_i = 5.0 \text{ s}$, and $\omega_i = -20 \text{ rpm} = -20(2\pi \text{ rad}/60 \text{ s}) = -\frac{2}{3}\pi \text{ rad/s}$, so $\omega_f = \omega_i + \alpha(t_f - t_i)$ becomes

$$0 \text{ rad/s} = \left(-\frac{2\pi}{3} \text{ rad/s}\right) + \alpha(5.0 \text{ s}) \Rightarrow \alpha = \frac{2\pi}{15} \text{ rad/s}^2$$

Having found I and α , we can now find the torque τ that will bring the balls to a halt in 5.0 s:

$$\tau = I_{\text{about cm}} \alpha = \left(\frac{2}{3} \text{ kg m}^2\right) \left(\frac{2\pi}{15} \text{ rad/s}^2\right) = \frac{4\pi}{45} \text{ N m} = 0.28 \text{ N m}$$

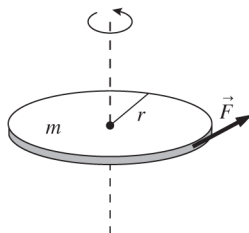
The magnitude of the torque is 0.28 N m, applied in the counterclockwise direction.

Problem 12-28

A 4.0 kg, 36-cm-diameter metal disk, initially at rest, can rotate on an axle along its axis. A steady 5.0 N tangential force is applied to the edge of the disk. What is the disk's angular velocity, in rpm, 4.0 s later?

12.28. Model: Model the disk as solid. The torque is constant so the angular acceleration is constant.

Visualize: The disk starts from rest, so $\omega_0 = 0$.



Known
 $m = 4.0 \text{ kg}$
 $r = 0.18 \text{ m}$
 $F = 5.0 \text{ N}$
 $\Delta t = 4.0 \text{ s}$
 $\omega_0 = 0$
Find
 ω_1

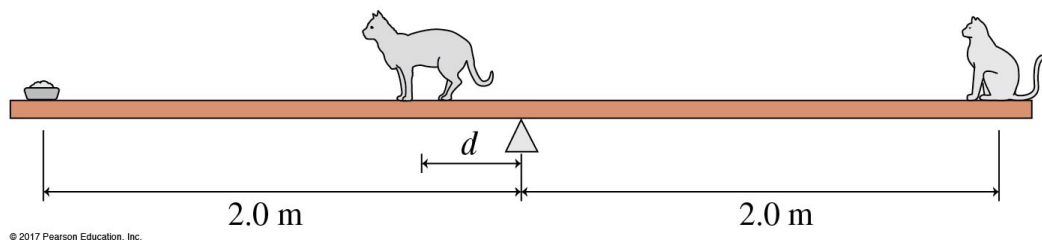
Solve:

$$\tau = I\alpha = I \frac{\Delta\omega}{\Delta t} \Rightarrow \Delta\omega = \omega_1 - \omega_0 = \omega_1 - 0 = \omega_1 = \frac{\tau \Delta t}{I} = \frac{rF\Delta t}{\frac{1}{2}mr^2} = \frac{F\Delta t}{\frac{1}{2}mr} = \frac{(5.0 \text{ N})(4.0 \text{ s})}{\frac{1}{2}(4.0 \text{ kg})(0.18 \text{ m})} = 55.6 \text{ rad/s} = 530 \text{ rpm}$$

Assess: 530 rpm is pretty fast but in the reasonable range.

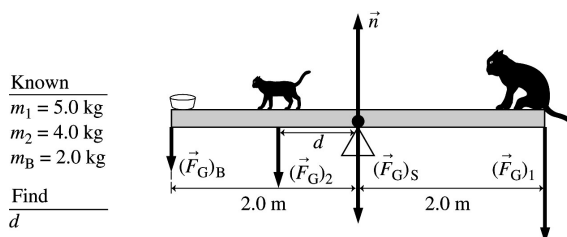
Problem 12-32

A 5.0 kg cat and a 2.0 kg bowl of tuna fish are at opposite ends of the 4.0-m-long seesaw of the figure below. How far to the left of the pivot point must a 4.0 kg cat stand to keep the seesaw balanced?



12.32. Model: The see-saw is a rigid body. The cats and bowl are particles.

Visualize:



Solve: The see-saw is in rotational equilibrium. Calculate the net torque about the pivot point.

$$\begin{aligned} \tau_{\text{net}} = 0 &= (F_G)_1(2.0 \text{ m}) - (F_G)_2(d) - (F_G)_B(2.0 \text{ m}) \\ m_2gd &= m_1g(2.0 \text{ m}) - m_Bg(2.0 \text{ m}) \\ d &= \frac{(m_1 - m_B)(2.0 \text{ m})}{m_2} = \frac{(5.0 \text{ kg} - 2.0 \text{ kg})(2.0 \text{ m})}{4.0 \text{ kg}} = 1.5 \text{ m} \end{aligned}$$

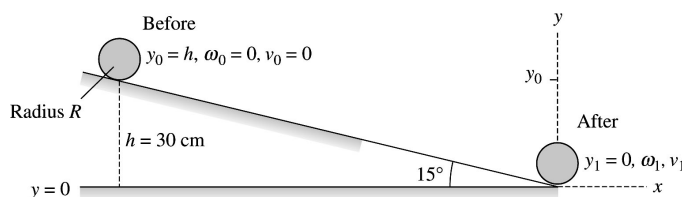
Assess: The smaller cat is close but not all the way to the end by the bowl, which makes sense since the combined mass of the smaller cat and bowl of tuna is greater than the mass of the larger cat.

Problem 12-36

A solid sphere of radius R is placed at a height of 30 cm on a 15° slope. It is released and rolls, without slipping, to the bottom. From what height should a circular hoop of radius R be released on the same slope in order to equal the sphere's speed at the bottom.

12.36. Model: The mechanical energy of both the hoop (h) and the sphere (s) is conserved. The initial gravitational potential energy is transformed into kinetic energy as the objects roll down the slope. The kinetic energy is a combination of translational and rotational kinetic energy. We also assume no slipping of the hoop or of the sphere.

Visualize:



The zero of gravitational potential energy is chosen at the bottom of the slope.

Solve: The energy conservation equation for the sphere or hoop $K_f + U_{gf} = K_i + U_{gi}$ is

$$\frac{1}{2}I(\omega_1)^2 + \frac{1}{2}m(v_1)^2 + mgy_1 = \frac{1}{2}I(\omega_0)^2 + \frac{1}{2}m(v_0)^2 + mgy_0$$

For the sphere, this becomes

$$\begin{aligned} \frac{1}{2}\left(\frac{2}{5}mR^2\right)\frac{(v_1)_s^2}{R^2} + \frac{1}{2}m(v_1)_s^2 + 0 \text{ J} = 0 \text{ J} + 0 \text{ J} + mgh_s \\ \Rightarrow \frac{7}{10}(v_1)_s^2 = gh \Rightarrow (v_1)_s = \sqrt{10gh/7} = \sqrt{10(9.8 \text{ m/s}^2)(0.30 \text{ m})/7} = 2.05 \text{ m/s} \end{aligned}$$

For the hoop, this becomes

$$\begin{aligned} \frac{1}{2}(mR^2)\frac{(v_1)_h^2}{R^2} + \frac{1}{2}m(v_1)_h^2 + 0 \text{ J} = 0 \text{ J} + 0 \text{ J} + mgh_{\text{hoop}} \\ \Rightarrow h_{\text{hoop}} = \frac{(v_1)_h^2}{g} \end{aligned}$$

For the hoop to have the same velocity as that of the sphere,

$$h_{\text{hoop}} = \frac{(v_1)_s^2}{g} = \frac{(2.05 \text{ m/s})^2}{9.8 \text{ m/s}^2} = 42.9 \text{ cm}$$

The hoop should be released from a height of 43 cm.

Problem 12-39

Vector $\vec{A} = 3\hat{i} + \hat{j}$ and vector $\vec{B} = 3\hat{i} - 2\hat{j} + 2\hat{k}$. What is the cross product $\vec{A} \times \vec{B}$?

12.39. Solve: $\vec{A} \times \vec{B} = (3\hat{i} + \hat{j}) \times (3\hat{i} - 2\hat{j} + 2\hat{k})$

$$\begin{aligned} &= 9\hat{i} \times \hat{i} - 6\hat{i} \times \hat{j} + 6\hat{i} \times \hat{k} + 3\hat{j} \times \hat{i} - 2\hat{j} \times \hat{j} + 2\hat{j} \times \hat{k} \\ &= 0 - 6\hat{k} + 6(-\hat{j}) + 3(\hat{k}) - 0 + 2\hat{i} = 2\hat{i} - 6\hat{j} - 9\hat{k} \end{aligned}$$

Problem 12-45

How fast, in rpm, would a 5.0 kg, 22-cm-diameter bowling ball have to spin to have an angular momentum of $0.23 \text{ kg m}^2/\text{s}$?

12.45. Model: The bowling ball is a solid sphere.

Solve: From Table 12.2, the moment of inertia about a diameter of a solid sphere is

$$I = \frac{2}{5}MR^2 = \frac{2}{5}(5.0 \text{ kg})(0.11 \text{ m})^2 = 0.0242 \text{ kg m}^2$$

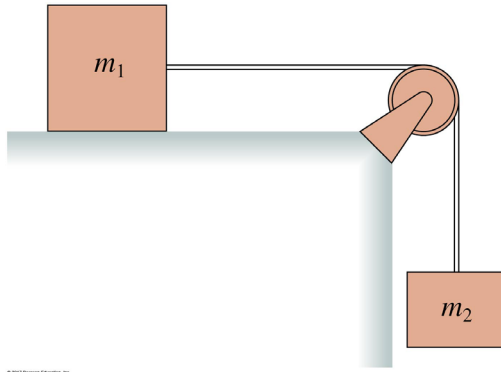
Require

$$\begin{aligned} L = 0.23 \text{ kg m}^2/\text{s} = I\omega = (0.0242 \text{ kg m}^2)\omega \\ \Rightarrow \omega = (9.5 \text{ rad/s}) \end{aligned}$$

In rpm, this is $(9.5 \text{ rad/s})\left(\frac{\text{rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right) = 91 \text{ rpm}$.

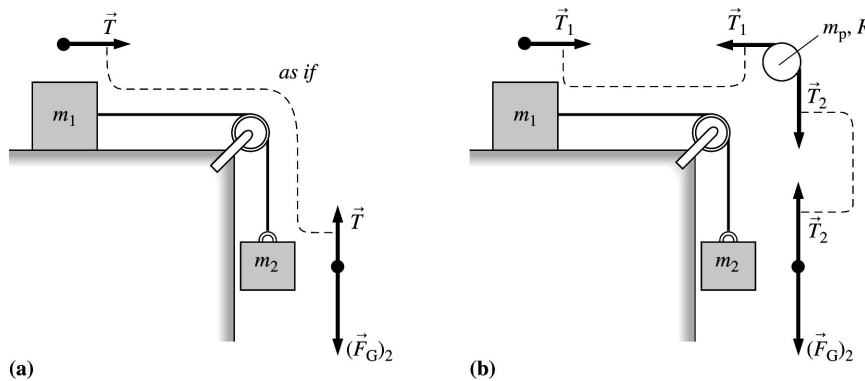
Problem 12-65

Blocks of mass m_1 and m_2 are connected by a massless string that passes over the pulley in the figure. The pulley turns on the frictionless bearings. Mass m_1 slides on a horizontal, frictionless surface. Mass m_2 is released while the blocks are at rest. **(a)** Assume the pulley is massless. Find the acceleration of mass m_1 and the tension in the string. This is a Chapter 7 review problem. **(b)** Suppose the pulley has mass m_p and radius R . Find the acceleration of m_1 and the tensions in the upper and lower portions of the string. Verify that the answers agree with part a if you set $m_p=0$.



12.65. Model: Assume the string does not slip on the pulley.

Visualize:



The free-body diagrams for the two blocks and the pulley are shown. The tension in the string exerts an upward force on the block m_2 , but a downward force on the outer edge of the pulley. Similarly the string exerts a force on block m_1 to the right, but a leftward force on the outer edge of the pulley.

Solve: **(a)** Newton's second law for m_1 and m_2 is $T = m_1 a_1$ and $T - m_2 g = m_2 a_2$. Using the constraint $-a_2 = +a_1 = a$, we have $T = m_1 a$ and $-T + m_2 g = m_2 a$. Adding these equations, we get $m_2 g = (m_1 + m_2) a$, or

$$a = \frac{m_2 g}{m_1 + m_2} \Rightarrow T = m_1 a = \frac{m_1 m_2 g}{m_1 + m_2}$$

(b) When the pulley has mass m , the tensions (T_1 and T_2) in the upper and lower portions of the string are different. Newton's second law for m_1 and the pulley are:

$$T_1 = m_1 a \quad \text{and} \quad T_1 R - T_2 R = -I \alpha$$

We are using the minus sign with α because the pulley accelerates clockwise. Also, $a = R\alpha$. Thus, $T_1 = m_1 a$ and

$$T_2 - T_1 = \frac{I}{R} \frac{a}{R} = \frac{aI}{R^2}$$

Adding these two equations gives

$$T_2 = a \left(m_1 + \frac{I}{R^2} \right)$$

Newton's second law for m_2 is $T_2 - m_2g = m_2a_2 = -m_2a$. Using the above expression for T_2 ,

$$a \left(m_1 + \frac{I}{R^2} \right) + m_2a = m_2g \Rightarrow a = \frac{m_2g}{m_1 + m_2 + I/R^2}$$

Since $I = \frac{1}{2}m_pR^2$ for a disk about its center,

$$a = \frac{m_2g}{m_1 + m_2 + \frac{1}{2}m_p}$$

With this value for a we can now find T_1 and T_2 :

$$T_1 = m_1a = \frac{m_1m_2g}{m_1 + m_2 + \frac{1}{2}m_p} \quad T_2 = a(m_1 + I/R^2) = \frac{m_2g}{(m_1 + m_2 + \frac{1}{2}m_p)} \left(m_1 + \frac{1}{2}m_p \right) = \frac{m_2(m_1 + \frac{1}{2}m_p)g}{m_1 + m_2 + \frac{1}{2}m_p}$$

Assess: For $m = 0$ kg, the equations for a , T_1 and T_2 of part (b) simplify to

$$a = \frac{m_2g}{m_1 + m_2} \quad \text{and} \quad T_1 = \frac{m_1m_2g}{m_1 + m_2} \quad \text{and} \quad T_2 = \frac{m_1m_2g}{m_1 + m_2}$$

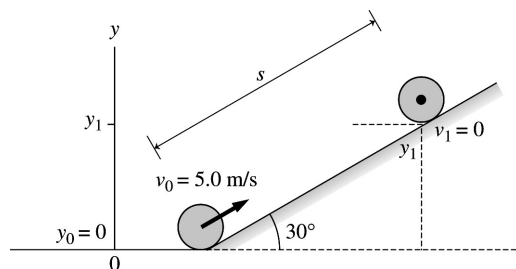
These agree with the results of part (a).

Problem 12-69

A hollow sphere is rolling along a horizontal floor at 5.0 m/s when it comes to a 30° incline. How far up the incline does it roll before reversing direction?

12.69. Model: Assume that the hollow sphere is a rigid rolling body and that the sphere rolls up the incline without slipping. We also assume that the coefficient of rolling friction is zero.

Visualize:



The initial kinetic energy, which is a combination of rotational and translational energy, is transformed in gravitational potential energy. We chose the bottom of the incline as the zero of the gravitational potential energy.

Solve: The conservation of energy equation $K_f + U_{gf} = K_i + U_{gi}$ is

$$\begin{aligned} \frac{1}{2}M(v_1)_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}(\omega_1)^2 + Mgy_1 &= \frac{1}{2}M(v_0)_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}(\omega_0)^2 + Mgy_0 \\ 0 \text{ J} + 0 \text{ J} + Mgy_1 &= \frac{1}{2}M(v_0)_{\text{cm}}^2 + \frac{1}{2} \left(\frac{2}{3}MR^2 \right) (\omega_0)_{\text{cm}}^2 + 0 \text{ J} \Rightarrow Mgy_1 = \frac{1}{2}M(v_0)_{\text{cm}}^2 + \frac{1}{3}MR^2 \frac{(v_0)_{\text{cm}}^2}{R^2} \\ \Rightarrow gy_1 &= \frac{5}{6}(v_0)_{\text{cm}}^2 \Rightarrow y_1 = \frac{\frac{5}{6}(v_0)_{\text{cm}}^2}{g} = \frac{5(5.0 \text{ m/s})^2}{6 \cdot 9.8 \text{ m/s}^2} = 2.126 \text{ m} \end{aligned}$$

The distance traveled along the incline is

$$s = \frac{y_1}{\sin 30^\circ} = \frac{2.126 \text{ m}}{0.5} = 4.3 \text{ m}$$

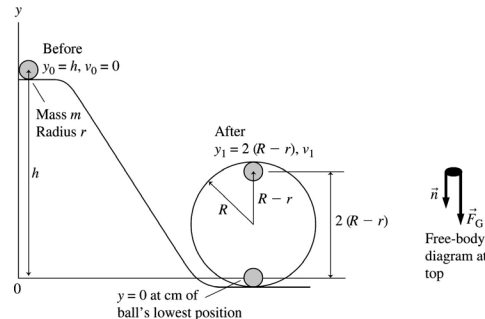
Assess: This is a reasonable stopping distance for an object rolling up an incline when its speed at the bottom of the incline is approximately 10 mph.

Problem 12-75

The marble rolls down the track shown in the figure and around a loop-the-loop of radius R . The marble has mass m and radius r . What minimum height h must the track have for the marble to make it around the loop-the-loop without falling off?

12.75. Model: Assume that the marble does not slip as it rolls down the track and around a loop-the-loop. The mechanical energy of the marble is conserved.

Visualize:



Solve: The marble's center of mass moves in a circle of radius $R - r$. The free-body diagram on the marble at its highest position shows that Newton's second law for the marble is

$$mg + n = \frac{mv_1^2}{R - r}$$

The minimum height (h) that the track must have for the marble to make it around the loop-the-loop occurs when the normal force of the track on the marble tends to zero. Then the weight will provide the centripetal acceleration needed for the circular motion. For $n \rightarrow 0$ N,

$$mg = \frac{mv^2}{(R - r)} \Rightarrow v_1^2 = g(R - r)$$

Since rolling motion requires $v_1^2 = r^2\omega_1^2$, we have

$$\omega_1^2 r^2 = g(R - r) \Rightarrow \omega_1^2 = \frac{g(R - r)}{r^2}$$

The conservation of energy equation is

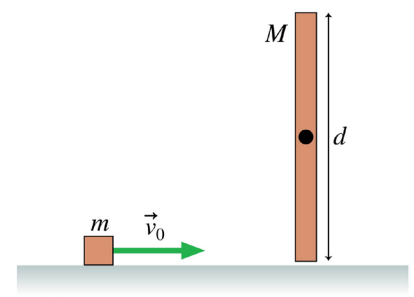
$$(K_f + U_{gf})_{\text{top of loop}} = (K_i + U_{gi})_{\text{initial}} \Rightarrow \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2 + mgy_1 = mgy_0 = mgh$$

Using the above expressions and $I = \frac{2}{5}mr^2$ the energy equation simplifies to

$$\frac{1}{2}mg(R - r) + \frac{1}{2}\left(\frac{2}{5}\right)mr^2\left(\frac{g(R - r)}{r^2}\right) + mg2(R - r) = mgh \Rightarrow h = 2.7(R - r)$$

Problem 12-89

The figure shows a cube of mass m sliding without friction at speed v_0 . It undergoes a perfectly elastic collision with the bottom tip of the rod of length d and mass $M = 2m$. The rod is pivoted about a frictionless axle through its center, and initially it hangs straight down and is at rest. What is the cube's velocity – both speed and direction – after the collision?



12.89. Model: Define the system as the rod and cube. Energy and angular momentum are conserved in a perfectly elastic collision in the absence of a net external torque. The rod is uniform.

Visualize: Please refer to Figure CP12.89.

Solve: Let the final speed of the cube be v_f , and the final angular velocity of the rod be ω . Energy is conserved, and angular momentum around the rod's pivot point is conserved.

$$E_i = E_f \Rightarrow \frac{1}{2}mv_0^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}I_{\text{rod}}\omega^2$$

$$L_i = L_f \Rightarrow mv_0\left(\frac{d}{2}\right) = mv_f\left(\frac{d}{2}\right) + I_{\text{rod}}\omega$$

This is two equations in the two unknowns v_f and ω . From Table 12.2,

$$I_{\text{rod}} = \frac{1}{12}Md^2 = \frac{1}{12}(2m)d^2 = \frac{1}{6}md^2$$

From the angular momentum equation,

$$v_0 = v_f + \frac{d}{3}\omega \Rightarrow \omega = \frac{3}{d}(v_0 - v_f)$$

Substituting into the energy equation,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{1}{6}md^2\right)\left(\frac{9}{d^2}\right)(v_0 - v_f)^2$$

$$v_0^2 = v_f^2 + \frac{3}{2}(v_0 - v_f)^2$$

$$0 = v_f^2 - \frac{6}{5}v_0v_f + \frac{1}{5}v_0^2$$

This is a quadratic equation in v_f . The roots are

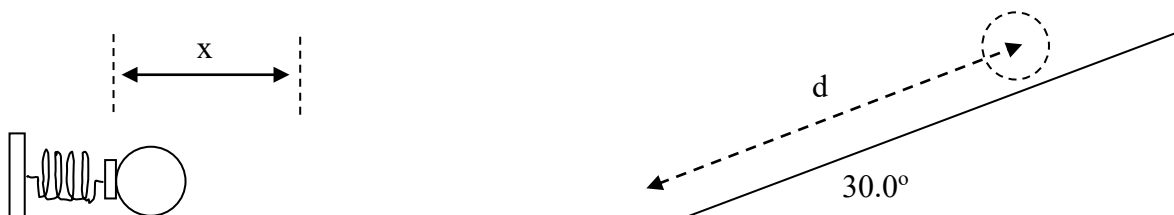
$$v_f = \frac{\frac{6}{5}v_0 \pm \sqrt{\left(\frac{6}{5}v_0\right)^2 - 4\left(\frac{1}{5}v_0^2\right)}}{2} = \frac{3}{5}v_0 \pm \frac{2}{5}v_0$$

$$= \begin{cases} \frac{1}{5}v_0 \\ v_0 \end{cases}$$

The answer $v_f = v_0$ means the ice cube missed the rod. So $v_f = \frac{1}{5}v_0$ to the right.

Problem 12-A


A solid steel ball of mass 0.50 kg and diameter 20 cm is held in place against a spring with spring constant $k = 100 \text{ N/m}$, compressing the spring a distance $x = 30 \text{ cm}$. The ball is then released from rest and rolls without slipping along a horizontal floor. It then makes a smooth transition to an inclined plane and rolls without slipping up the plane as shown in the figure below.



a) How far up the incline plane (i.e. what is d in the figure) does the ball roll before coming to rest?

$$W_{nc} = 0 \rightarrow E_f = E_i$$

$$\frac{1}{2} K X_f^2 + mg y_f + \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 = \frac{1}{2} K X_i^2 + mg y_i + \frac{1}{2} m v_i^2 + \frac{1}{2} I \omega_i^2 \rightarrow mg y_f = \frac{1}{2} K X_i^2$$

$$y_f = \frac{K X_i^2}{2mg} = \frac{(100 \text{ N/m})(0.30 \text{ m})^2}{2(0.50 \text{ kg})(9.80 \text{ m/s}^2)} = 0.92 \text{ m}$$


$$d = 0.92 \text{ m} / \sin 30^\circ$$

b) How fast is the ball rolling (i.e. what is the speed of the ball's center of mass) when it has rolled 1.2 m up the inclined plane?

$$d = 1.84 \text{ m}$$

$$\frac{1}{2} K X_f^2 + mg y_f + \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 = \frac{1}{2} K X_i^2 + mg y_i + \frac{1}{2} m v_i^2 + \frac{1}{2} I \omega_i^2$$

$$mg y_f + \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 = \frac{1}{2} K X_i^2 \rightarrow \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 = \frac{1}{2} K X_i^2 - mg y_f$$

$$I = \frac{2}{5} m r^2 \quad \left. \begin{array}{l} \\ \omega = v/r \end{array} \right\} \frac{1}{2} m v_f^2 + \frac{1}{2} \left(\frac{2}{5} m r^2 \right) \left(\frac{v}{r} \right)^2 = \frac{1}{2} K X_i^2 - mg y_f$$

$$\frac{7}{10} m v_f^2 = \frac{1}{2} K X_i^2 - mg y_f \rightarrow v_f = \sqrt{\frac{10}{7} \frac{(\frac{1}{2} K X_i^2 - mg y_f)}{m}}$$

Problem 2 using $y_f = 0.60 \text{ m} \rightarrow v_f = 2.1 \text{ m/s}$

Problem 12-B

In the figure below, a solid ball rolls smoothly from rest (starting at height $H = 6.0 \text{ m}$) until it leaves the horizontal section at the end of the track at height $h = 2.0 \text{ m}$. How far horizontally from point A does the ball hit the floor?



\Rightarrow first get speed at bottom of ramp from conservation of energy:

$$y=0 \quad K_f + U_f = K_i + U_i \rightarrow K_f = U_i$$

$$\frac{1}{2} I_{cm} \omega_f^2 + \frac{1}{2} M v_f^2 = M g y_i$$

$$\frac{1}{2} \left(\frac{2}{5} M R^2 \right) \left(\frac{v}{R} \right)^2 + \frac{1}{2} M v^2 = M g y_i \rightarrow \frac{1}{5} M v^2 + \frac{1}{2} M v^2 = M g y_i$$

$$\frac{7}{10} M v^2 = M g y_i \rightarrow v = \sqrt{\frac{10}{7} g y_i} = \sqrt{\left(\frac{10}{7} \right) (9.8 \text{ m/s}^2) (4.0 \text{ m})} = 7.48 \text{ m/s}$$

now use projectile motion equations to get horizontal distance:

$$\text{time in air} = 0.645 \rightarrow \text{horizontal distance} = 4.8 \text{ m}$$

Problem 4

Problem 12-C

At the instant the displacement of a 2.00 kg object relative to the origin is

$$\vec{d} = (2.00\text{ m})\hat{i} + (4.00\text{ m})\hat{j} - (3.00\text{ m})\hat{k}$$

its velocity is $\vec{v} = -(6.00\text{ m/s})\hat{i} + (3.00\text{ m/s})\hat{j} + (3.00\text{ m/s})\hat{k}$

and it is subject to a force $\vec{F} = (6.00\text{ N})\hat{i} - (8.00\text{ N})\hat{j} + (4.00\text{ N})\hat{k}$. Find (a) the acceleration of the object, (b) the angular momentum of the object about the origin, (c) the torque about the origin acting on the object, and (d) the angle between the velocity of the object and the force acting on the object.

$$(a) \quad \vec{a} = \vec{F}/m = \frac{(6.00\text{ N})\hat{i} - (8.00\text{ N})\hat{j} + (4.00\text{ N})\hat{k}}{2.00\text{ kg}}$$

$$\vec{a} = (3.00\text{ m/s}^2)\hat{i} - (4.00\text{ m/s}^2)\hat{j} + (2.00\text{ m/s}^2)\hat{k}$$

$$(b) \quad \vec{\ell} = \vec{r} \times \vec{p} = \vec{r} \times (m\vec{v}) = m(\vec{r} \times \vec{v})$$

$$\vec{r} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2.00\text{ m} & 4.00\text{ m} & -3.00\text{ m} \\ -6.00\text{ m/s} & 3.00\text{ m/s} & 3.00\text{ m/s} \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} 4.00\text{ m} & -3.00\text{ m} \\ 3.00\text{ m/s} & 3.00\text{ m/s} \end{vmatrix} - \hat{j} \begin{vmatrix} 2.00\text{ m} & -3.00\text{ m} \\ -6.00\text{ m/s} & 3.00\text{ m/s} \end{vmatrix} + \hat{k} \begin{vmatrix} 2.00\text{ m} & 4.00\text{ m} \\ -6.00\text{ m/s} & 3.00\text{ m/s} \end{vmatrix}$$

$$= (21.0\text{ m}^2/\text{s})\hat{i} + (12.0\text{ m}^2/\text{s})\hat{j} + (30.0\text{ m}^2/\text{s})\hat{k}$$

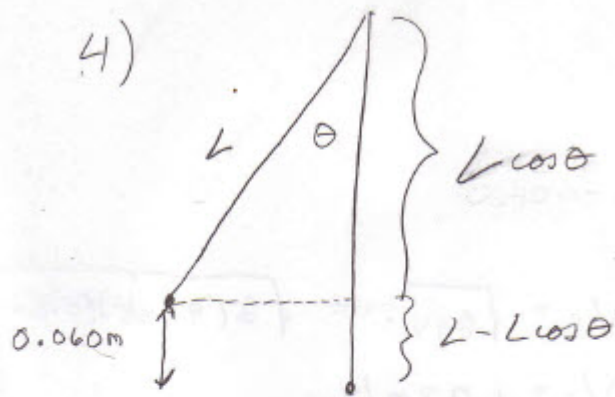
$$\vec{\ell} = m(\vec{r} \times \vec{v}) = (2.00\text{ kg})(\vec{r} \times \vec{v}) = (42.0\text{ kg m}^2/\text{s})\hat{i} + (24.0\text{ kg m}^2/\text{s})\hat{j} + (60.0\text{ kg m}^2/\text{s})\hat{k}$$

$$(c) \quad \vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2.00\text{ m} & 4.00\text{ m} & -3.00\text{ m} \\ 6.00\text{ N} & -8.00\text{ N} & 4.00\text{ N} \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} 4.00\text{ m} & -3.00\text{ m} \\ -8.00\text{ N} & 4.00\text{ N} \end{vmatrix} - \hat{j} \begin{vmatrix} 2.00\text{ m} & -3.00\text{ m} \\ 6.00\text{ N} & 4.00\text{ N} \end{vmatrix} + \hat{k} \begin{vmatrix} 2.00\text{ m} & 4.00\text{ m} \\ 6.00\text{ N} & -8.00\text{ N} \end{vmatrix}$$

$$\vec{\tau} = (-8.00\text{ N m})\hat{i} - (26.0\text{ N m})\hat{j} - (40.0\text{ N m})\hat{k}$$

4)



$$L - L \cos \theta = 0.060 \text{ m}$$

$$L(1 - \cos \theta) = 0.060 \text{ m}$$

$$1 - \cos \theta = 0.060 \text{ m} / L \rightarrow \cos \theta = 1 - 0.060 \text{ m} / L$$

$$\cos \theta = 1 - 0.060 \text{ m} / 0.400 \text{ m} = \underline{\underline{0.85}}$$

$$\boxed{\theta = 32^\circ}$$